Impedance and Collective Effects in Future Light Sources

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In future ring-based light sources, the combination of low emittance and high current will mean that collective effects will be important

Outline of Talk

- Intra-beam scattering
- Touschek lifetime
- Longitudinal broad-band impedance
  -- Coherent synchrotron radiation
  -- Insertion transition impedance calculations
- Instabilities
- Conclusions

As specific example illustrating the above topics I will use the case of PEP-X, a study effort of a group led by Y. Cai; see White Paper for more details
## Selected Parameters for PEP-X

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, $E$</td>
<td>4.5</td>
<td>GeV</td>
</tr>
<tr>
<td>Circumference, $C$</td>
<td>2199.</td>
<td>m</td>
</tr>
<tr>
<td>Average current, $I$</td>
<td>1.5</td>
<td>A</td>
</tr>
<tr>
<td>Bunch population, $N_b$</td>
<td>2.18</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>Number of bunches, $M$</td>
<td>3154</td>
<td></td>
</tr>
<tr>
<td>Relative rms energy spread, $\sigma_{\rho}$</td>
<td>1.14</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Rms bunch length, $\sigma_{z0}$</td>
<td>3.0</td>
<td>mm</td>
</tr>
<tr>
<td>Horiz. emittance parameter, $\epsilon_{x00}$</td>
<td>85.7</td>
<td>pm</td>
</tr>
<tr>
<td>Horiz. radiation damping time, $\tau_x$</td>
<td>13.5</td>
<td>ms</td>
</tr>
<tr>
<td>Long. radiation damping time, $\tau_p$</td>
<td>7.2</td>
<td>ms</td>
</tr>
</tbody>
</table>

Note that the nominal horizontal emittance $\epsilon_{x0} = \epsilon_{x00}/(1+\kappa)$, with $\kappa$ the $x$-$y$ coupling parameter.
Intra-Beam Scattering (IBS) Calculations

IBS describes multiple Coulomb scattering that (in electron machines) leads to an increase in all bunch dimensions and in energy spread

Assume coupling dominated: \( \epsilon_y = \kappa \epsilon_x \)

Steady-state IBS emittance and energy spread:

\[
\epsilon_x = \frac{\epsilon_{x0}}{1 - \tau_x / T_x} \quad \text{and} \quad \sigma_p^2 = \frac{\sigma_{p0}^2}{1 - \tau_p / T_p}
\]

Local IBS growth rates \( \delta(1/T_x) \), \( \delta(1/T_p) \), are functions of beam and lattice parameters; their average around the ring are the growth rates \( 1/T_x \), \( 1/T_p \)

We follow the Bjorken-Mtingwa (BM) method; solution involves (i) integration at every lattice element, (ii) averaging around the ring, (iii) solving the above two equations simultaneously

Due to small impact parameter events, the tails of distributions are not Gaussian; our Coulomb log reflects this (see Kubo and Oide); for PEP-X, \( \log \approx 13 \)
Simplified Model of IBS

(K. Bane, EPAC02)

Longitudinal growth rate:

\[
\frac{1}{T_p} \approx \frac{r_e^2 c N_b (\log \gamma)}{16 \gamma^3 \epsilon_x^{3/4} \epsilon_y^{3/4} \sigma_z \sigma_p^3} \left< \sigma_H g(a/b) (\beta_x \beta_y)^{-1/4} \right> = \delta(1/T_p)
\]

\[
\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{\mathcal{H}_x}{\epsilon_x}, \quad a = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_x}{\epsilon_x}}, \quad b = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_y}{\epsilon_y}}
\]

\[
g(\alpha) = \alpha^{(0.021 - 0.044 \ln \alpha)}
\]

Transverse growth rate:

\[
\frac{1}{T_x} = \frac{\sigma_p^2}{\epsilon_x} \left< \mathcal{H}_x \right> \frac{1}{T_p}
\]

\[
\frac{1}{T_x} = \frac{\sigma_p^2}{\epsilon_x} \left< \mathcal{H}_x \delta(1/T_p) \right>
\]

Valid for \( a, b \ll 1 \), “high energy approximation”
Solution for PEP-X

• For PEP-X two modes of operation:
  (1) nominal—adjust $\kappa$ so that steady-state $\varepsilon_y = 8$ pm (diffraction limited at 1 angstrom)
  (2) running as FEL in one straight section, so $\kappa = 1$

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\epsilon_{x0}$ [pm]</th>
<th>$\epsilon_x$ [pm]</th>
<th>$\epsilon_y$ [pm]</th>
<th>$\sigma_p$ [$10^{-3}$]</th>
<th>$\sigma_z$ [mm]</th>
<th>$T$ [min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>.049</td>
<td>82.</td>
<td>164.</td>
<td>8.0</td>
<td>1.20</td>
<td>3.16</td>
<td>29.</td>
</tr>
<tr>
<td>1.</td>
<td>43.</td>
<td>69.</td>
<td>69.</td>
<td>1.17</td>
<td>3.08</td>
<td>92.</td>
</tr>
</tbody>
</table>

Table. Results for flat and round-beam cases at nominal current $I = 1.5$ A: coupling parameter, nominal (zero-current) emittance, and steady-state beam properties. The last column gives the Touschek lifetime (discussed below).

• Note: almost no growth in $p$ or $z$

• In nominal configuration $T_x^{-1} = 24.7$ s$^{-1}$, $T_p^{-1} = 9.5$ s$^{-1}$ (simplified model gets $T_x^{-1} = 22.9$ s$^{-1}$, $T_p^{-1} = 12.0$ s$^{-1}$)
Accumulated IBS Growth Rates

Accumulated growth rates in $p$, $x$; $\mathcal{H}_x$ optics function
Correlation between $H_x$ and $\delta(1/T_p)$ in PEP-X

$H_x$ and $\delta(1/T_p)$ over one TME and one DBA arc of PEP-X

Note the anti-correlation of the two functions in the DBA cells

With no correlation but "same" lattice parameters, $1/T_x$ would be twice as large
Touschek Lifetime

- Touschek effect concerns large, single Coulomb scattering events where energy transfer from transverse to longitudinal leads to immediate particle loss.

- Calculation follows method of Brueck, as modified by Piwinski; valid for flat beams.

- Number of particles in bunch decays as:

\[ N_b = \frac{N_{b0}}{1 + t/T} \]

- Inverse of Touschek lifetime (assuming \( \varepsilon_y = \kappa \varepsilon_x \)) is given by:

\[ \frac{1}{T} = \frac{r_e^2 e N_b}{8 \pi \beta^3 \gamma^5 \sigma_z \kappa^{1/2} \varepsilon_x^2} \left( \frac{\beta_x^{3/2} \sigma_x^2 C(\epsilon_m)}{\beta_y^{1/2} \tilde{\sigma}_x^3 \epsilon_m} \right) \]

\[ C(\epsilon_m) = -\frac{3}{2} e^{-\epsilon_m} + \int_{\epsilon_m}^{\infty} \left( 1 + \frac{3\epsilon_m}{2} + \frac{\epsilon_m}{2} \ln \frac{u}{\epsilon_m} \right) e^{-u} \frac{du}{u} \]

\[ \epsilon_m = \frac{\beta_x \sigma_x^2 \delta_m}{\gamma^2 \epsilon_x \tilde{\sigma}_x^2} \]
Momentum acceptance due to linear optics for PEP-X. The locations of the TME and DBA arcs are also indicated in the figure.
Touschek Lifetime Results

• Results for the IBS-determined steady-state beam sizes are: $T = 29$ min (nominal, flat beam), $T = 92$ min (round beam)

• Impedance generated potential well distortion may help (~25%?)

• Stable top-up injection needed

Accumulation around the ring of the Touschek growth rate in the nominal PEP-X configuration.
**Increase at very low Emittance?**

If the emittance becomes small enough, \( T \) should start increasing again. Can we go to this regime?

Touschek lifetime as function of \( \varepsilon_x \), with \( \varepsilon_y, \sigma_p \) fixed. This is not a self-consistent calculation. (The lattice is for a slightly earlier version of PEP-X.)
Longitudinal Impedance Calculations

- A bottom-up approach to the impedance—i.e. starting from drawings of vacuum chamber components—has had a few successes in existing machines, e.g. the SLC damping rings, Dafne.

- For PEP-X, without an actual vacuum chamber design available, we are developing a straw man design, inspired by objects in other machines, such as PEP-II

  Sources include: RF cavities, BPM’s, wiggler transitions, undulator transitions, resistive wall, coherent synchrotron radiation (CSR)

For the microwave instability, generate:

(i) a pseudo-Green function wake representing the ring—to be used in simulations ($\sigma_z = .5 \text{ mm}$; nominal is $3 \text{ mm}$) and reaching to $60 \text{ mm}$ behind the bunch

(ii) an impedance budget—to assess relative importance of contributors

People involved in 3D code development and impedance calculation include L.-Q. Lee, C.-K. Ng, L. Wang, L. Xiao
Bunch Lengthening in DAΦNE (M. Zobov)

Typical Measured Bunch Distributions

**e+ Ring**

- 19.5 mA
- 19.2 mA
- 19.0 mA
- 18.5 mA
- 18.0 mA
- 17.5 mA
- 17.0 mA
- 16.5 mA
- 16.0 mA
- 15.5 mA
- 15.0 mA
- 14.5 mA
- 14.0 mA
- 13.5 mA
- 13.0 mA
- 12.5 mA
- 12.0 mA
- 11.5 mA
- 11.0 mA

**e- Ring**

- 20 mA
- 19.5 mA
- 19.0 mA
- 18.5 mA
- 18.0 mA
- 17.5 mA
- 17.0 mA
- 16.5 mA
- 16.0 mA
- 15.5 mA
- 15.0 mA
- 14.5 mA
- 14.0 mA
- 13.5 mA
- 13.0 mA
- 12.5 mA
- 12.0 mA
- 11.5 mA
- 11.0 mA

**FWHM/2.3548 [cm]**

- Measurements 2000
- Simulation 1998
- Measurements 2004

Comparison with Simulations

M. Zobov et al., e-Print: physics/0312072
Selected PEP-X Impedance Sources

Selected impedance objects included in our straw man PEP-X design. Note: the fundamental mode fields are shown in the RF cavity.
Impedance Budget

<table>
<thead>
<tr>
<th>Object</th>
<th>Single Contribution</th>
<th>Total Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_{loss}$ [V/pC]</td>
<td>$R$ [Ω]</td>
</tr>
<tr>
<td>RF cavity</td>
<td>.92</td>
<td>30.4</td>
</tr>
<tr>
<td>Undulator taper (pair)</td>
<td>.06</td>
<td>3.2</td>
</tr>
<tr>
<td>Wiggler taper (pair)</td>
<td>.43</td>
<td>21.4</td>
</tr>
<tr>
<td>BPMs</td>
<td>.013</td>
<td>.6</td>
</tr>
<tr>
<td>Bellows slots</td>
<td>.00</td>
<td>.0</td>
</tr>
<tr>
<td>Bellows masks</td>
<td>.005</td>
<td>.2</td>
</tr>
<tr>
<td>Resistive wall wake</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Impedance budget for PEP-X, giving the loss factor, and the effective resistance and inductance of the various objects in the ring. The results are at nominal bunch length $\sigma_z = 3$ mm.
Pseudo-Green Function Wake

Pseudo-Green function wake representing the PEP-X ring: wake of a $\sigma_z = .5 \text{ mm}$ bunch (tentative)

Inductive in character

Haissinski solution, giving the steady-state bunch shape. Bunch length is 25% above nominal length.

(G. Stupakov)
CSR Wake

Free space solution: wake of Gaussian bunch, \( \mathcal{W}_{gz} = \frac{Z_0 c}{2\pi} w_{gz} R^{-2/3} \sigma_z^{-4/3} \) (Murphy, et al; Derbenev, et al)

Parallel plate shielding can also be included. Shielding is effective for \( \sigma_z/a > (a/R)^{1/2} \) (Warnock)

Effect in rectangular beam pipe can also be found (Stupakov, Kotelnikov)
CSR in PEP-X

(G. Stupakov)

CSR wake in PEP-X with rectangular chamber and with parallel plates

Total wake in PEP-X for .5 mm bunch (red), and without CSR contribution (blue)

Note: in PEP-X $\sigma_z R^{1/2}/a^{3/2} = 2.2$ (for $\sigma_z = .5$ mm, $a = 12.5$ mm, $R = 40$ m—DBA bends) $\Rightarrow$ expect CSR shielded

Vacuum chamber shape
Insertion Transition Calculations

From “Impedance calculation for the NSLSII storage ring,” A. Blednykh

With the insertion gap becoming ever smaller, the insertion region becomes a dominating part of the ring impedance.

Insertion transitions tend to be long, gradually tapered, and 3D => it is very challenging to obtain the wakefield for a short bunch.

- Study dependence of result on configuration/truncation of problem
- Impedance scaling of small angle tapers
Transition Impedance Study

- Numerically study: (1) difference when a symmetric taper is modeled as an in-taper, a collimator or a cavity, and (2) effect of truncation on the result

- Perform 2D study using ECHO2D—I. Zagorodnov’s time domain code

- PEP-X may have 8 damping wiggler down-taper pairs (of length 25 meters each) and 8 up-taper pairs (of length 250 meters each)

- Taper angle ~6 deg, connecting $a_y = 7.5$ mm to $b = 48$ mm; in reality 3D object
Types of Tapers (Cylindrically Symmetric)

\[ v = c \]

\[ Z_{in} = Z_1 - Z_s, \quad Z_{out} = Z_1 + Z_s, \quad Z_s = \left( \frac{Z_0}{2\pi} \right) \ln(b/a) \]  
(see e.g. R. Gluckstern, B. Zotter, LEP-Note 613, 1988)

• For symmetric collimator or cavity, with long central region, \( Z_{sym} \approx Z_{in} + Z_{out} = 2(Z_{in} + Z_s) \), or \( W_{sym} \approx 2(W_{in} + W_s) \), with \( W_s = -Z_s c \lambda_z \)
In-Taper Calculation

\[
\begin{align*}
\text{In-Taper Calculation} \\
\begin{array}{c}
\downarrow \\
\text{b} \\
\uparrow \\
\text{v= c} \\
\downarrow \\
\text{a} \\
\end{array}
\end{align*}
\]

\[
a = 7.5 \text{ mm}, \quad b = 48 \text{ mm}, \\
\text{taper is } \sim 6 \text{ deg}, \quad \sigma_z = .5 \text{ mm}
\]

\[
\frac{v}{c}
\]

\text{Wake of symmetric collimator using in-taper calculation}
Symmetric Tapered Collimator--Varying Length

\[ a = 7.5 \text{ mm}, \quad b = 48 \text{ mm}, \]
\[ \text{taper is } \sim 6 \text{ deg}, \quad \sigma_z = 0.5 \text{ mm} \]

Catch-up distance \( z_{cu} = \frac{a^2}{2\sigma_z} \sim 56 \text{ mm} \)

*Wake of symmetric, tapered collimator as function of length. Result of in-taper calculation is in blue*

• To reach the asymptotic result to 10 mm behind the bunch, need \( L > \sim 10z_{cu} \)
Tapered Cavity Wake vs Length

\( a = 7.5 \text{ mm}, \ b = 48 \text{ mm}, \) taper is \( \sim 6 \text{ deg}, \ \sigma_z = 0.5 \text{ mm} \)

Catch-up distance \( z_{cu} = (b-a)^2/2\sigma_z \sim 1.6 \text{ m} \)

Wake of \( \sigma_z = 0.5 \text{ mm} \) cavity with tapered sidewalls, for several values of cavity length \( L \). The in-taper result is given in blue.

- To reach the asymptotic result to 10 mm behind the bunch, need \( L >> 10z_{cu} \)

Conclusion:

For accurate taper wake calculation may need to consider full-length structure
Impedance Scaling for Small Angle Tapers
(G. Stupakov, SLAC-TN-10-001, Feb 2010)

• Consider a structure geometry whose walls \( r(z) \) describe a smooth function

• If one scales only the \( z \) dimension by \( \lambda \), G. Stupakov has shown, from the parabolic equation for fields at high frequency, that \( Z(\omega) = Z_\lambda(\lambda \omega) \)

• For small angle tapers the scaling is true at all frequencies—including low frequencies, where Yokoya’s formula applies: \( Z(\omega) = -i \omega L \), with \( L \) proportional to the slope of the taper

• This implies that for small angle the bunch wake \( W(s) = W_\lambda(s/\lambda)/\lambda \), if \( \sigma_{z\lambda} = \sigma_z/\lambda \)

• Similar scaling works for transverse wake: \( W_x(s) = W_{x\lambda}(s/\lambda) \)

• For 3D calculation implies reduction in required computer resources of \( \lambda^4 \)
Wake from In-Taper Calculations

With $\lambda = .5$, can reduce time and memory requirements in 3D calculation by a factor 16, with $\lambda = .33$ the factor is 81.

Wake of $\sigma_z = .5 \text{ mm bunch (blue)}$; and the scaled calculation $\lambda = .5 \text{ (red)}$, and $\lambda = .33 \text{ (yellow)}$; the bunch shape is also shown (green).

- We are currently numerically testing the impedance scaling for specific 3D taper examples.
Instabilities in PEP-X

G. Stupakov, L. Wang

Microwave:
Can cause $\sigma_p, \sigma_z$ to increase, saw-tooth instability, heating, $\sigma_y$ growth (Dafne). With current PEP-X impedance we find 25% $\sigma_z$ increase (potential well distortion) but no threshold until $I > 8$ A

Transverse Single Bunch:
Can cause beam to be lost. When including only the resistive wall impedance (dominant in the insertion devices), the threshold is 1.5 times the nominal current. Other impedance objects will reduce this value.

Since resistive wall wake $\propto a^{-3}$, ($a$ is aperture) this is a serious limitation to reducing the aperture of the insertions. (How about cooling beam pipe? For Cu at 77 K, wake is reduced by factor 2.5, at 2 K by 13.)

Multibunch Transverse:
Including only the resistive wall wake, which often dominates the multibunch transverse instability, the calculation yields a growth time of .14 ms, or 19 turns. Good feedback will be needed.

Fast Ion:
Multi-particle tracking shows that the instability is strong, though with a compromise between vacuum level and bunch gap size, the growth rate can be kept to a level that are manageable with feedback.
VFP Solver for Microwave Instability

- Vlasov-Fokker-Planck solver based on the work of Warnock and Ellison with an improved interpolation scheme
- Solves Haissinski’s integral equation and use its solution as a starting distribution
- Handle singular wakefields, such as CSR wakefield in vacuum or resistive-wall wakefield
- CSR shielding in two parallel plates based on work of Murphy, Krinsky, and Gluckstern
- Many other models: Q=1 resonator, pure inductance or resistance, or any numerical wakefield, for example, wakefield of the SLC damping ring
**CSR Instability:** For 3 mm Gaussian bunches, estimated threshold of bunch current for the CSR-driven instability is 0.64 mA, corresponding to a total current of 2.0 A. The simulation, using a VFP solver and parallel plates model, shows a higher threshold near 3.0 A.

\[ I_{b}^{th} = \frac{\sqrt{2\pi \gamma \alpha \sigma_{\delta}^{2} \sigma_{z}}}{2^{8/3} g} \]
Q = 1 Resonator

$N_s$—time in units of synchrotron period

Resonator frequency:
$\omega_0 \sigma_z/c = 1$

Normalized current:

$$S = \frac{e^2 N}{2\pi E\nu_s \sigma_p \omega_0 R/Q}$$

Damping used: $\omega_s \tau_d = 800$
Conclusions

• In future high current, low emittance ring-base light sources, like PEP-X, collective effects are important, especially intrabeam scattering, the Touschek effect, and the transverse single bunch instability.

• For PEP-X, with $\varepsilon_y = 8 \text{ pm}$, IBS yields a doubling of $\varepsilon_x$. The Touschek lifetime is very short, 30 min. The threshold to the microwave instability, however, appears to be far above the nominal current.

• Have investigated asymptotic behavior of impedance for tapers like those for the damping wigglers of PEP-X

  --To reach asymptotic result flat regions of pipe need to be $\sim 10$ times catch up distance

  --Scaling theorem for small angle tapers seems to work. It allows one to speed up a 3D calculation by $\lambda^4$, where $\lambda$ is the scaling parameter—an impossible problem may then become tractable