Free electron laser simulations without the slowly-varying envelope approximation

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ICFA Advanced Beam Dynamics Workshop on Future Light Sources
Stanford, CA
March 4th, 2010
Outline

• What is the SVEA? Why apply it?

• How do we get around it?
  ✤ The unaveraged FEL code Aurora

• What might violate it?
  ✤ Fast-varying seed radiation (HHG)
  ✤ Fast-varying output radiation (high $\rho$ regime)
  ✤ Fast longitudinal variations in e-beam profile

• Conclusions
SVEA overview

• Light sources generally produce radiation with an amplitude that does not vary on the scale of a wavelength.

• Allows simplification of wave equation:

\[
\left( \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \tilde{E}(z, t) = \frac{1}{\epsilon_0 c^2} \frac{\partial}{\partial t} \tilde{J}(z, t)
\]

\[
\tilde{E}(z, t) = E(z, t) \exp[ik(z - ct)] \quad \tilde{J}(z, t) = J(z, t) \exp[ik(z - ct)]
\]
SVEA overview

\[ \left( \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) + 2ik \left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \right] E(z, t) = \frac{1}{\epsilon_0 c^2} \left( \frac{\partial}{\partial t} - ick \right) J(z, t) \]

- Slowly-varying envelope approximation:
  \[ \left| \frac{\partial^2 E}{\partial z^2} \right| \ll 2k \left| \frac{\partial E}{\partial z} \right| \quad \left| \frac{\partial^2 E}{\partial t^2} \right| \ll 2ck \left| \frac{\partial E}{\partial t} \right| \quad \left| \frac{\partial J}{\partial t} \right| \ll ck |J| \]

  \[ \left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) E(z, t) = -\frac{1}{2\epsilon_0 c} J(z, t) \]

- Much simpler evolution of radiation field.
Wiggler period averaging

- FEL gain stems from resonant processes
The code Aurora

• One-dimensional model
  ✤ No radiation diffraction
  ✤ No electron beam emittance
  ✤ Constant, matched transverse profiles
    (cross-section $\Sigma_b$)

• Multi-frequency
  ✤ E-field recorded on many points
    per resonant wavelength $\lambda_0$

S. Bajlekov et al., FEL’09 MOPC42
FEL simulations without the SVEA

Svetoslav Bajlekov – 04 Mar 2010
Radiation field

\[
\left( \frac{\partial^2}{\partial z'^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t'^2} \right) A_x(z, t) = -\frac{1}{\epsilon_0 c^2} J_x(z, t)
\]

ref frame moving at \( c \)
\[
z' = z \\
\frac{\partial}{\partial z} \rightarrow \frac{\partial}{\partial z'} - \frac{1}{c} \frac{\partial}{\partial t'}
\]
\[
t' = t - \frac{z}{c} \\
\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t'}
\]

backward wave
\[
\left( \frac{\partial^2}{\partial z'^2} - \frac{2}{c} \frac{\partial^2}{\partial z' \partial t'} \right) A_x(z, t) = -\frac{1}{\epsilon_0 c^2} J_x(z, t)
\]

FFT
\[
\frac{\partial}{\partial z} \tilde{A}_x(z, k) = \frac{1}{2ik\epsilon_0 c^2} \tilde{J}_x(z, k)
\]
Current density, Macroparticles

\[ J_x(z, \theta) = \frac{e \sqrt{2} a_u}{\Sigma_b} \cos(k_u z) \sum_{i=1}^{N_{\text{part}}} \frac{w_i}{\gamma_i} C(\theta - \theta_i) \]

\[ \frac{\partial \gamma_i}{\partial z} = - \frac{\sqrt{2} e a_u}{\gamma_i m c^2} \cos(k_u z) E(z, \theta_i) \]

\[ \frac{\partial \theta_i}{\partial z} = - \frac{k_0}{2 \gamma_i^2} \left( 1 + a_u^2 \left[ 1 + \cos(2k_u z) \right] \right) \]
When might we violate the SVEA?

1. When input/seed field has wavelength-scale amplitude variation.
   - HHG seeding

2. When output field has wavelength-scale amplitude variation
   - Cooperation length \( \approx \) wavelength; or \( \rho \approx 1 \)

3. Highly modulated electron bunch
CASE 1: HHG seeds
The three-step model

a. Tunnel ionization into continuum
b. Electron pulled away by laser field
c. Electron driven back towards ion
d. Recombination & emission of radiation


harmonic cut-off energy

\[ I_p + 3.17U_p \]
\[ U_p = \frac{E_0^2}{4\omega^2} \]
Temporal/spectral structure

Simulate “toy” HHG seed

$\lambda_0 = 10.67$ nm (75$^{th}$ harm)

$T_L = 800$ nm / c

50 as FWHM

Intensity (a.u.)

$n_{\text{harm}}$

FEL simulations without the SVEA

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Simulate “toy” HHG seed

Parameters:
- $\lambda_u = 0.02$ m
- $a_u = 0.82$
- $\gamma = 1250$
- $\sigma_{\gamma}/\gamma = 0 / 0.1$ %
- $I = 10 / 100$ kA
- $\sigma_r = 100$ $\mu$m
- $P_{\text{seed}} = 580$ kW

$\lambda_0 = 10.67$ nm
(75$^{th}$ harm)

$T_L = 800$ nm / $c$

50 as FWHM

Intensity (a.u.)

Spectrum
SVEA makes no difference here

• Two ways to explain this
Only resonant component of seed matters

- **a.** Unfiltered seed
- **b.** Only fundamental wavelength

Energy within window (J)

\[
z(z_u)
\]
Only resonant component of seed matters

- a. Unfiltered seed
- b. Only fundamental wavelength
- c. Only resonant component
FEL equations (I)

Emission

- Radiation emission governed by wave equation:
  \[
  \left( \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) A_x(z, t) = -\frac{1}{\epsilon_0 c^2} J_x(z, t)
  \]

- Evolution equation:
  \[
  2ik \frac{\partial \tilde{A}}{\partial \tilde{z}} = -\frac{1}{\epsilon_0 c^2} \tilde{J} - \frac{d^2 \tilde{A}}{\partial \tilde{z}^2} + \frac{2}{c} \frac{\partial^2 \tilde{A}}{\partial \tilde{z} \partial \tilde{t}}
  \]

\[
A(\tilde{z}, \tilde{t}) = \tilde{A}(\tilde{z}, \tilde{t}) e^{-ikc\tilde{t}}
\]
FEL equations (II)

Microbunching

• Energy modulation

\[
\frac{d\gamma}{dt} = \frac{e}{mc} \vec{E} \cdot \vec{\beta}
\]

\[
\alpha = \frac{e}{mc^2} \frac{a_w [JJ]}{\sqrt{2}}
\]

\[
\frac{d\gamma}{dz} = 2\frac{\alpha}{\gamma} \cos(k_w z) E_0(z - ct) \cos[k(z - ct) + \psi]
\]

• All contributions

\[
\frac{d\gamma}{d\zeta} = \frac{\alpha ck}{\gamma_0 k_w} \left\{ A_0(\theta - \zeta) [\cos(\theta + \psi) + \cos(\theta + \psi - 2\zeta)] + A'_0(\theta - \zeta) [\sin(\theta + \psi) + \sin(\theta + \psi - 2\zeta)] \right\}
\]
FEL equations (II)

• Energy modulation

\[ \sigma_2 = 1.0 \sigma_1 \]
\[ \sigma_2 = 1.1 \sigma_1 \]
\[ \sigma_2 = 2.0 \sigma_1 \]

\[ \frac{d\gamma}{d\zeta} = \frac{\alpha ck}{\gamma_0 k_w} \{ A_0(\theta - \zeta)[\cos(\theta + \psi) + \cos(\theta + \psi - 2\zeta)] + A'_0(\theta - \zeta)[\sin(\theta + \psi) + \sin(\theta + \psi - 2\zeta)] \} \]

magnitude of correction terms for single spike of length \( \sigma_{1,2} \)

\[ \alpha = \frac{e}{mc^2} \frac{a_w[JJ]}{\sqrt{2}} \]
Possible violation of SVEA: High-$\rho$ case

• SASE emission characterized by spikes of duration equal to the cooperation length

\[ L_{\text{coop}} = \frac{L_G}{\lambda w} \lambda = \frac{\lambda}{4\pi \sqrt{3\rho}} \]

• When $\rho \approx 0.05 \Rightarrow L_{\text{coop}} \approx \lambda$

• Correspondingly, $\rho$ is FEL gain bandwidth
Gain curves: $\rho = 0.005$
Gain curves: $\rho = 0.005$
Gain curves: $\rho = 0.005$
Gain curves: $\rho = 0.02$
Gain curves: $\rho = 0.02$

**Diagram Description:**
- **Axes:**
  - Y-axis: Output (a.u.)
  - X-axis: Frequency $\omega/\omega_0$
- **Graph:**
  - Red line: Aurora
  - Blue line: Perseo
- **Label:**
  - "normalized" indicating that the graph has been normalized.

**Notes:**
- The graph shows the gain curves for different frequencies normalized to their respective maxima.
- The X-axis is labeled with values $0.8, 0.85, 0.9, 0.95, 1, 1.05, 1.1, 1.15, 1.2$.
- The Y-axis is labeled with values $0, 0.5, 1, 1.5, 2, 2.5, 3 \times 10^{14}$.
Gain curves: $\rho = 0.05$
Gain curves: $\rho = 0.05$

Normalized graph showing outputs for Aurora and Perseo codes.
Gain curves: $\rho = 0.10$
Gain curves: $\rho = 0.10$
Broadband seeds in High-\(\rho\) case

Unclear whether presence of additional harmonics within gain bandwidth affects gain in fundamental
Coherent spontaneous emission

• Coherent emission due to short-scale longitudinal variations in bunch profile

• Not directly related to SVEA, but not simulated by conventional codes
Coherent spontaneous emission

- Coherent emission due to short-scale longitudinal variations in bunch profile
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SVEA overview

Aurora code

HHG seeding

High $\rho$ regime
Conclusions

• Worth considering situations that may challenge slowly-varying envelope approximation in FEL codes
• Unaveraged 1-D code Aurora works without SVEA; allows us to test such situations
• HHG seeding is simulated correctly in conventional codes despite temporal structure
• When Pierce parameter $\rho \gtrsim 0.02$, gain curves altered ...situation not readily encountered
Acknowledgments

Oxford

- Simon Hooker
- Riccardo Bartolini

LBNL

- Bill Fawley
- Carl Schroeder
Thanks!