Beam Optics & Dynamics Studies for LHC

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What am I doing at ETH Zurich?

*Simulation of low energy (1 eV – 1 keV) electron beam – matter interaction for SEM critical dimension nanometrology.*

*Simulation of SEM image formation.*

*Development of fast CD extraction and surface reconstruction methods from SEM images.*
Simulation of multi-bunch instabilities

Transverse multi-bunch instabilities, high energy proton beams (LHC)

Beam optics & design

LHC injector complex upgrade, beam optics design PS2 transfer line

Beam dynamics simulation with MAD-X

Abort gap cleaning in the LHC (simulation & benchmark measurements in the CERN SPS)
Transverse multi-bunch instabilities, high energy proton beams (LHC)

SIMULATION MULTI-BUNCH INSTABILITIES
Simulating LHC beam stability

- PhD title: *Simulation of transverse multi-bunch instabilities of proton beams in LHC*
- Development of a particle tracking code that is able to
  - *study the stability behavior of the multiple bunches of LHC (2808)*
  - *model the long-range impedance effects correctly (resistive wall)*
  - *do the computation sufficiently fast!*

LHC (Large Hadron Collider) at CERN

Cross section variation and/or finite wall conductivity

Vacuum pipe with cross section variation

Collective Effects
(caused by fields acting back on the beam)

Instabilities
(Motion of Particles becomes unstable)
Simulating beam stability: Wake Summation Problem

- Summation of wake field contributions of ~3000 bunches over ~100 turns

For each bunch at each time step:
Recompute wake function of all preceding bunches and all preceding turns!

\[ W_{\perp}(\tau) \propto 1/\sqrt{\tau} \]

\[ W_{\perp}(\tau + \Delta \tau) = \gamma \cdot W_{\perp}(\tau) \]

\Rightarrow Fast summation via HH Convolution

\[ W_{\perp}(\gamma) \propto \exp(j \omega_1 \gamma) \]

\[ W_{\perp}(\tau + \Delta \tau) = \exp(j \omega_1 \Delta \tau) \cdot W_{\perp}(\tau) \]

\Rightarrow Time evolution and summation using Phasors

= Resonator Model \((R, Q, \omega_r)\)
Simulating beam stability: Wake Summation Problem

Effect of preceding bunches of preceding turns on current bunch at current turn

\[ \Delta x^{(j)}_{n} = \sum_{i=0}^{j-1} \frac{\langle x \rangle_{n}^{i}}{\sqrt{(j-i) \cdot \tau_{bu.c.}}} + \sum_{k=n-n_{\text{mem}}}^{n-1} \sum_{i=0}^{N_{b}-1} \frac{\langle x \rangle_{k}^{i}}{\sqrt{(n-k) \cdot \tau_{rev.}} + (j-i) \cdot \tau_{bu.c.}} \]

- Sum over preceding bunches at current turn
- Sum over preceding bunches over previous turns

\[ \mathcal{O}(N_{b}^{2}) \]

Brute force summation results in computation times in the order of days/weeks!
Simulating beam stability: Wake Summation Solution

- Discrete FFT Convolution algorithm used to speed up the computation significantly ($N^2 \rightarrow N \log N$)

Analogy between wake sum and (discrete) convolution:

$$\Delta x^w_n = \sum_{k=0}^{n-1} \frac{\langle x \rangle_k}{\sqrt{(n-k) \cdot \tau_{rev.}}} = \sum_{k=0}^{n-1} g(k) \cdot f(n-k) \quad \text{(single bunch case)}$$

Continuous Convolution

$$h(t) = g \ast f \equiv \int_{-\infty}^{\infty} g(\tau) f(t-\tau) \, d\tau$$

Discrete Convolution

$$h(n) = g \ast f = \sum_{k=0}^{N-1} g(k) f(n-k) - \sum_{k=0}^{N-1} g_k f_{n-k}$$

Convolution Theorem

$$\mathcal{F}[f \ast g] = \mathcal{F}[f] \mathcal{F}[g] \quad \text{or} \quad f \ast g = \mathcal{F}^{-1}[\mathcal{F}[f] \mathcal{F}[g]]$$

Compute wake sum via the FFT convolution in frequency domain:

$$\Delta x^w_n = h_n = (g \ast f)_n = \sum_{k=0}^{N-1} g_k f_{n-k} \quad \xrightarrow{\text{FFT}} \quad G_j F_j = H_j$$

Direct Summation

$$(2n_{\text{mem}} + 1) N_b^2 + N_b$$

FFT Convolution

$$(n_{\text{mem}} + 2) 4N_b \log 4N_b + (5n_{\text{mem}} + 5) N_b$$

Brings computation times from days/weeks down to hours!
Simulating beam stability: Impedance/Wake Models

Knowledge of correct impedance model critical for machine performance!

Frequency range interesting for LHC
Simulating beam stability: Impedance/Wake Models

- Classical resistive wall impedance model was found to be an inappropriate description for the physical situation in the LHC.

- For one new model, which fits well with measurement the corresponding wake function was computed and implemented in the code MTRISIM.

$$Z_{m=1}^{\perp, \text{thick,ibp}}(\omega) = (1 + j \ \text{sgn} \ \omega) \frac{c \mu_0 L}{2 \pi b^2} \frac{1}{-j + \text{sgn} \ \omega \left(1 + b \sqrt{\frac{\sigma c \mu_0}{2 \mu_r}} \sqrt{|\omega|}\right)}$$

**Fourier Transform (not straightforward!)**

$$W_{m=1}^{\perp, \text{thick,ibp}}(t) = + \frac{cL}{\pi^{3/2} b^3} \sqrt{\frac{\mu_0 \mu_r}{\sigma c}} \cdot \frac{1}{\sqrt{|t|}}$$

**classic thick wall wake function**

- $$- \exp \left[ \frac{4 \mu_r}{b^2 \sigma c \mu_0} |t| \right] \frac{2cL \mu_r}{b^4 \pi \sigma_c} \cdot \left(1 - \text{Erf} \sqrt{\frac{4 \mu_r}{b^2 \sigma c \mu_0} |t|}\right)$$

**correction term due to inclusion of inductive bypass**
Simulating beam stability: The code MTRISIM

- Classical tracking code
  Transfer matrices + non-linear kicks

- Rigid bunch approximation

- Wake function approximation
  \[ W_{pot}(\tau) \approx W(\tau) \]
  \[ W_{pot}^{||}(\tau) = \int_0^\infty dt \ W_{||}(t) \ \lambda(\tau - t) \]
  \[ W_{pot}^{\perp}(\tau) = \frac{1}{\xi} \int_0^\infty dt \ W_{\perp}(t) \ \xi(\tau - t) \ \lambda(\tau - t) \]

- Lumped impedance approximation
  Impedance modelled by 1 kick per turn\(^a\)

- Non-circular vacuum chamber geometries: Yokoya factors used!
  \[ W_{pot}^{||}(x, y, \bar{x}, \bar{y}, s) \approx x \ W_{pot}^{||}(x, s) + \bar{x} \ W_{pot}^{\perp}(\bar{x}, s) \]
  \[ W_{pot}^{\perp}(x, y, \bar{x}, \bar{y}, s) \approx y \ W_{pot}^{\perp}(x, s) + \bar{y} \ W_{pot}^{\perp}(\bar{y}, s) \]

\[ \epsilon_{new} = \epsilon + \delta \ \epsilon \]
\[ \bar{x}_{new} = \bar{x} + \delta \ \bar{x} \]
\[ \bar{\epsilon}_{new} = \bar{\epsilon} + \delta \ \bar{\epsilon} + e V^{RF} \cdot \sin(\varphi_s + \omega^{RF} \ t) \]

\[ M = \begin{pmatrix} \sqrt{\frac{\beta_{i+1}}{\beta_i}} \cos(\mu(\epsilon)) & \sqrt{\beta_{i+1} \beta_i} \sin(\mu(\epsilon)) \\ -\frac{1}{\sqrt{\beta_{i+1} \beta_i}} \sin(\mu(\epsilon)) & \sqrt{\frac{\beta_{i+1}}{\beta_i}} \cos(\mu(\epsilon)) \end{pmatrix} \]

\[ \mu(\epsilon) = \mu_0 \left[ 1 + \frac{\xi - \eta}{E_0} \cdot \epsilon \right] \]

\[ \delta \bar{x} = \begin{pmatrix} 0 \\ \Delta \epsilon \end{pmatrix} \]

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Simulating beam stability: LHC Simulations

Amplitude growth of individual bunches vs. Turns
Machine Resistance only, LHC Injection Energy, nominal Intensity

Machine Resistance + Collimators, LHC Injection Energy, nominal Intensity
Coupled-bunch mode spectra vs. Turns
Machine Resistance + Collimators + HOMs and Octupoles
LHC Top Energy, ultimate Intensity

Non-linearities (octupoles) provide amplitude dependent detuning and stabilize the beam. Important result for stability and performance of the LHC machine!
Simulating beam stability: Summary

- **Objectives**
  - study the stability behavior of the multiple bunches of LHC (2808)
  - model the long-range impedance effects correctly (resistive wall)
  - do the computation sufficiently fast!

- **Achievements**
  - Efficient implementation of wake summation via discrete FFT convolution algorithm approach ($N^2 \rightarrow N \log N$).
    
    Brings computation time from days/weeks down to hours!
  - Resistive wall impedance models in a new parameter regime, corresponding wake function computed and used in simulation.
  - Code benchmarked with measurements in CERN SPS.
  - Simulation of LHC. Present octupole design will provide enough Landau damping to stabilize beam at top energy.

- **Follow-up work, latest developments**
  - Fast wake summation algorithm used for CLIC damping ring studies at Cockcroft Institute, Daresbury, UK:
  - MTRISIM recently added to CERN BE-ABP code-pool for thorough re-study of LHC impedance issues (Elias Metral, Benoit Salvant, Nicolas Mounet)
LHC injector complex upgrade, beam optics design PS2 transfer line

BEAM OPTICS & DESIGN
Several work packages in the context of
- LHC commissioning preparation
- (new & proposed) accelerator optics design
- design & commissioning of beam transfer systems
- LHC injection optics (transfer lines TI2, TI8)
- Dispersion matching knobs LHC
- Solenoid coupling compensation LHC
- Beam Commissioning of the SPS LSS6 Extraction and TT60
- Beam Commissioning of the SPS-to-LHC Transfer Line TI 2
- LHC sector test: Magnet quench test
- PS2 Injection, Extraction and Beam Transfer Concepts
- Initial design of the PS2-SPS transfer line

Coupling compensation by skew quads

\[
\tilde{c}_{\text{sol}} = -\frac{i}{4\pi B \rho} \left( \frac{\beta_y^s}{\beta_x^s} \pm \frac{\beta_x^p}{\beta_y^p} \right)
\]

\[
\sum_m c_{\text{skew}}^+ = \sum_m \frac{1}{2\pi} \sqrt{\beta_x^m \beta_y^m} \cdot k_s^m \cdot e^{i(\mu_x^m + \mu_y^m)} = c_{\text{sol}}^+
\]

\[
\sum_m c_{\text{skew}}^- = \sum_m \frac{1}{2\pi} \sqrt{\beta_x^m \beta_y^m} \cdot k_s^m \cdot e^{i(\mu_x^m - \mu_y^m)} = c_{\text{sol}}^-
\]

**Resonant orbit bumps**

**Localized dispersion wave**

\[
D(s) = \frac{\sqrt{\beta(s)}}{2 \sin \pi Q} \int \frac{\sqrt{\beta(t)}}{\rho(t)} \cos \left( |\Phi(t) - \Phi(s)| - \pi Q \right) dt
\]

**Figures:**
- Graphs showing resonant orbit bumps and localized dispersion waves.
Several work packages in the context of:

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Beam optics design: PS2 – SPS Transfer Line Design

Design Goal
- Shortest possible transfer line accomplishing all required features:
  - Matched optics at both ends (PS2, SPS)
  - Meet space/geometry requirements
  - Low $\beta$ insertion (stripping foil)
  - Emittance exchange scheme
  - Switch to experimental area
Beam optics design: PS2 – SPS TL Design Approach

- Start from SPS injection optics and calculate Twiss parameters in *backward* manner through SPS injection region, TT10 and the transfer line

- **Match optics** \((\beta, \alpha, D, D')\) at virtual PS2 extraction point.
  Assumptions extraction point: \(\beta_x = 6.25\, m\), \(\beta_y = 36.0\, m\), \(\alpha, D, D' = 0.0\)
  ➔ Get dispersion function \(D\) under control! Achromats!

- **Match space/geometry requirements**
  - Transfer Line defines location of PS2!
  - 15m separation between PS2 tunnel and existing tunnels (radiation protection)
  ➔ Length limits for Transfer Line! MAD-X matching with survey coordinates!

- Include: **low \(\beta\) insertion** for ion stripping (Pb), **emittance exchange** scheme, switch to experimental area, …

- **Beam momentum:** 50 GeV/c

- **Dipole Magnets:** \(l = 3.0\, m\)
  \(1.6\, T\) field strength
  \(1/\rho = 9.59\, mrad/m\) bending angle

- **Quadrupole Magnets:** \(l = 1.75\, m\)
  \(16\, T/m\) field gradient
  \(k_{\text{max}} = 0.0959\)
Beam optics design: PS2 – SPS TL Design

- Matching section (with low-β insertion) near SPS
- 2 bending sections (opposite direction) as *achromats* \((D=D'=0 \text{ at each end})\)

Matching section:
Used to match SPS to achromat entry conditions

Achromat 1 + Achromat 2:
Used to match to PS2 extraction conditions

\(\alpha_{\text{bend}} = -320 \text{ mrad}\)

\(\alpha_{\text{bend}} = +160 \text{ mrad}\)
Beam optics design: PS2 – SPS TL Design

MAD-X optics matching: Achromat dipole strengths define location of PS2 machine (survey coordinates)
Beam optics design: PS2 TL Optics $\beta$– functions

**Diagram:**
- SPS injection region
- Matching section
- Achromat 1
- Achromat 2
- PS2 extraction section
- PS2 LSS

**Graph:**
- $\beta_x$ (Low-$\beta$)
- $\beta_y$

**Legend:**
- $TT12$ Layout 4.6.1.4 match $TT12$-PS2

**Axes:**
- $s (m)$
- $\beta_x$, $\beta_y$ (m)
Beam optics design: PS2 TL Optics Hor. Dispersion

\[ J_D = \frac{D^2 + (\beta D' + \alpha D)^2}{2 \beta} \]

\[ \Delta J_D = \Theta \cdot (\beta D' + \alpha D) \]

dispersion action

dispersion modifier at bend with angle \( \Theta \)
Possible solution:
3 additional cells (2+1 because of asymmetry) with skew quads inbetween the two achromats (because dispersion free there!)

Fits space requirements if injection/extraction areas in PS2 are swapped.

Note:
Used MAD-X PTC!
Beam optics design: PS2 TL Design Summary

Objectives
- feasibility study of transfer line connecting PS2 - SPS
- meet tight space/geometry requirements
- Low-\(\beta\) insertion, emittance exchange scheme, ...

Achievements
- Design goals can be achieved with this initial layout, in particular the tight space and geometry requirements!
- Optics design using MAD-X (with PTC), plus a lot of experienced/educated guesses!
- Optics matching with survey coordinates (user defined macro function matching in MAD-X).

Follow-up work, latest developments
- Hessler, C; Bartmann, W; Benedikt, M; Goddard, B; Meddahi, M; Uythoven, J: "Transfer Lines to and from PS2"; Proc. of 1st IPAC, Kyoto, Japan, 23 - 28 May 2010, sLHC-PROJECT-Report-0043; CERN-sLHC-PROJECT-Report-0043
- Hessler, C; Goddard, B; Meddahi, M: "PS2 Transfer Lines"; sLHC Project Note 0013, 2010-02-04
- CERN management has stopped PS2 project in favor of refurbishing the old PS (Chamonix Meeting 2010).
Abort gap cleaning in the LHC
(simulation & benchmark measurements in the CERN SPS)

BEAM DYNAMICS SIMULATION MAD-X
Abort Gap Cleaning

- LHC beam dump kicker system
  *field rise time* 3 μs

- Particle-free *abort gap of 3 μs* needed for clean beam dump process

- Particles filling abort gap see *sweeping kicker field* and may be lost at various positions in the machine, possibly causing a *magnet quench*.

- Assuming quench limit of 4 mJ/cm³, maximum abort gap population tolerated at 7 TeV corresponds to 
  \[1.7 \times 10^7 \text{ p+/bunch} \text{ (2} \times 10^6 \text{ p+/m)}\]

- Abort gap population from uncaptured beam or longitudinal diffusion processes:
  \[\sim 3 \times 10^{10} \text{ p+/100 ns (450 GeV)}\]
  \[\sim 3 \times 10^8 \text{ p+/100 ns (7 TeV)}\]

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Abort Gap Cleaning – Principle

- **Particle tunes off from nominal tune due to:**
  - Chromaticity
  - Non-linearities (amplitude-dependent tune change)

- **Tracking studies** to investigate cleaning efficiency dependent on non-linear effects.

- **Tune modulated** kicker signal
- **Resonantly excite** increasing transverse particle oscillations
- **Loose particles on aperture limits** (ideally collimators) in controlled way
Abort Gap Cleaning – SPS Simulations with MADX

- **Motivation:** Assess feasibility and performance of LHC abort gap cleaning system
- **Objective:** Benchmark simulation against measurements to gain confidence for LHC
  - Investigate influence of non-linearities (octupoles) and chromaticity on cleaning speed and efficiency
  - Experiment with different excitation programs (fixed freq., sweep, freq. modulation)

- MAD-X full 6D element-by-element tracking with aperture.
- **Additionally:** Implementation of time (i.e. turn by turn) varying kicks in MADX
- **Simulation:**
  - Set optics (e.g. nominal SPS optics, LHC beam & working point) and beam parameters
  - Vary $\xi$ chromaticity
  - Vary $k_3$ octupole strength (simulate non-linearities)
  - Different transverse damper excitation programs
  - Record loss rate, integrated loss and loss pattern

- **Used MADX model:**
  SPS 2007 sequence/strength/aperture set-up
  
  $\sigma_{Ap/p} = +/- 2.37 \times 10^{-3}$
  
  central tunes: $Q_H = 26.13$  
  $Q_V = 26.18$
  
  transverse emittances: $\varepsilon_n = 3 \mu m$

  $\sigma_v = 2.96 \text{ mm @ TIDV.11892 (} \beta = 81 \text{ m) @ 26 GeV/c}$
  
  Beam should be mainly lost at TIDV with half gap of 20.4 mm corresponding to 6.9 $\sigma$
Abort Gap Cleaning – SPS Simulations with MADX

Chromaticity low \((\xi_H=0.02, \xi_V=0.03)\)
Octupoles OFF

Loss time \(\approx 5-7 \text{ ms} (250 \text{ turns})\)

Chromaticity medium \((\xi_H=0.02, \xi_V=0.43)\)
Octupoles OFF

Loss time \(\approx 25-30 \text{ ms} (1100 \text{ turns})\)

Full longitudinal motion simulated, i.e. RF on
Abort Gap Cleaning – SPS Measurements

Chromaticity low ($\xi_H=0.02$, $\xi_V=0.03$), Octupoles off
fixed frequency excitation

Loss signal BLM @ TIDV

SPS.BCTDC.41435

Loss time $\approx 5$ ms (220 turns)

Optimized excitation frequency
Abort Gap Cleaning – SPS Measurements

Chromaticity medium \((\xi_H=0.02, \xi_V=0.43)\), Octupoles ON: LOF +20.0

gap cleaning efficiency

Loss signal BLM @ TIDV

SPS.BCTDC.41435

Loss time \(\approx 20\) ms (870 turns)
Abort Gap Cleaning – Optimizing frequency programs

- **Excitation conditions:**
  - Vertical damper V1 (BDV 214.55) on, V2 (BDV 221.76) off, Amplitude: 2.89 $\mu$rad per turn
  - Excitation at 1-q$_{frac}$ (for better efficiency due to hardware limitations) at $t = 1000$ms in the cycle.

- Programmable arbitrary waveform generator used (Agilent 33250 A).
  - Limitation 65536 points, use of low-pass filter to smooth waveform. Max. ~4000 turns excitation.

- The revolution frequency at 26 GeV is 43.347282 kHz
  - Excitation was always at $(1-q_{frac}) \times f_{rev}$, i.e. around 35 kHz

- Programs used for MD:
  - Fixed frequency
    - Good for constant tune
  - Swept frequency
    - Tune changes with amplitude
  - Modulated frequency
    - (FM, modulation depth, modulation freq.)
    - Efficient in presence of chromaticity
  - FM on a carrier that is itself swept
    - Octupoles + chromaticity
Abort Gap Cleaning – Summary

Objectives

- Transient simulations using MAD-X tracking, full 6D + time-varying kicks
- Benchmark measurements in the SPS

Achievements

- Tool to test & develop best cleaning strategy for the LHC

Follow-up work, latest developments

- “LHC Abort Gap Monitoring and Cleaning”; Meddahi, M; Gianfelice-Wendt, E et al.; 1st IPAC, Kyoto, Japan, 23 - 28 May 2010; CERN-ATS-2010-124

Chromaticity medium ($\xi_H=0.02$, $\xi_V=0.43$)

Octupoles ON

Simulated and measured loss rates in very good agreement

Loss time $\approx 25-30$ ms (1100 turns)

Close collaboration with BI and RF group!
FINAL WRAP-UP
Final Wrap-up

- Simulation of transverse multi-bunch instabilities (LHC)  
  physical modeling, algorithm design, simulation development, beam measurements

- Beam optics & design, PS2 transfer line optics  
  linear optics design, optics matching, MAD-X (PTC), commissioning with beam

- Beam tracking with MAD-X (Abort Gap Cleaning)  
  simulation & benchmark measurements