Magnetic lenses play a critical role in accelerator applications. Strength & tunability desirable.

Accelerator applications → transport line
• Magnetic lens for collimation, focusing
• Focal length ~ gamma / (dB/dr)
• High gradient → compact applications
• High gradient → high energy applications
• Ideal: tunable, radially symmetric
• Active Plasma Lens meets these criteria
Discharge current flowing through a capillary: linear B-gradient for uniform current distribution

Ampere’s law

\[
2\pi r B(r) = \mu_0 \int_0^r 2\pi r' J(r') dr'
\]

\[
\int_0^R 2\pi r J(r) dr = I_0
\]

Uniform current

\[
J(r) = J_0
\]

\[
J_0 = \frac{I_0}{\pi R^2}
\]

\[
B(r) = \frac{\mu_0 I_0}{2\pi R^2} r
\]

\[
\frac{\partial B}{\partial r} = \frac{\mu_0 I_0}{2\pi R^2}
\]
Active plasma lens an “old” concept for ion beams. Attractive due tunability, symmetry, and strength

Active Plasma Lens
- Introduced 1950s (ion beams) \(^{\text{Panofski et al. RSI 1950}}\)
- Radial symmetric focusing
- Tunable, up to ~1 kA (“active”)  
- Gradients >3000 T/m

Field gradient
\[
\frac{\partial B}{\partial r} = \frac{\mu_0 I}{2\pi R^2}
\]

Strength parameter
\[
k = \frac{q}{m_0 \gamma c} \frac{\partial B}{\partial r}
\]

Focal length
\[
F = \frac{1}{kL}
\]

Example
D=0.5mm, L=6cm  
F=2cm @ 500 MeV  
I=430 A (1400 T/m)

Example
D=0.5mm, L=6cm  
F=25cm @ 10 GeV  
I=695 A (2200 T/m)

1 mrad & 25cm → \(\sigma=250 \, \mu\text{m}\)
Experimental demonstration on Laser-Plasma Accelerated electron beams in 2015

van Tilborg et al. PRL 115, 184802 (2015)
Energy-dispersed beam size diagnostic reveals oscillations within plasma lens

- D 0.25mm capillary, L=33mm, tape protection
- Agreement with simulation
- @300A → 3000 T/m
- 2\textsuperscript{nd} and 3\textsuperscript{rd} oscillation inside lens observed

van Tilborg et al. PRL 115, 184802 (2015)
Transverse scan useful to measure focusing gradient

- Good technique regardless of imaging geometry
- D 0.5mm capillary, L=15mm, 30mm drift, no tape protection
- Under investigation: measured current ~360 Amps
- Cap wider (on average) by 21%?
Where active plasma lenses can provide unique solutions:

1. Ultra-relativistic e-beams

**Quadrupole doublet: Asymmetric focusing**

For 300 MeV e-beam
- Solenoid (2 T, L=20 cm): F=500 cm
- Quad doublet (500 T/m, L=3 cm): F=13 cm
- Active plasma lens (2000 T/m, L=3 cm): F=1.7 cm

For 10 GeV e-beam
- Quad doublet (500 T/m, L=3 cm): F=4.5 m
- Active plasma lens (2000 T/m, L=6 cm): F=0.28 m

(also of interest to ion beams)  

Panofski et al. RSI 1950, Boggasch et al. Proceedings EPAC ‘92
Where active plasma lenses can provide unique solutions: 2. Rapid capture for compactness, mitigation ε growth

- Compact (staging)
- Quick capture → emittance mitigation for $\Delta \gamma / \gamma \sim \%$
- Quick capture → matching in undulator for larger $\Delta \gamma / \gamma$

\[ \varepsilon_n = \sqrt{\varepsilon_0^2 + \sigma_\theta^4 \sigma_\gamma^2 \sigma^2 / \gamma^2} \]

**Steinke et al. Nature 2016**

**Migliorati et al. PRSTAB 2013**

**Barber et al. Proceedings AAC 2016**
Possible limitation: emittance degradation from beam-driven wakefields for dense e-beams


\[
\frac{(E_r - B_\varphi)(r,\zeta)}{E_0} = \frac{\pi (k_p L)}{\pi^2 - k_p^2 L^2} n_b \left\{ \frac{1}{k_p} \sin \left[ k_p \left( \frac{\zeta - L}{2} \right) \right] + \frac{L}{\pi} \cos \left[ \frac{\pi \zeta}{L} \right] \right\} \times \left[ -\frac{2r}{r_b^2} + 2k_p I_0'(k_p r) K_2(k_p r_b) \right]
\]
Possible limitation: emittance degradation from beam-driven wakefields for dense e-beams

To minimize wakefield effect:
- Let e-beam divergence more
- Operate at highest current (shortest cap) → reduce relative effect
- Emittance degradation \( \sim \frac{1}{\gamma} \)
- Away from resonant density (weak effect, low density most practical)
- Wakefields play role for “>20pC sub-GeV <100\mu m” beams
Possible limitation: emittance degradation from non-uniform current

Radial current distribution dominated by temperature
Towards peak of current pulse: more current on-axis!

Classical conductivity
(resistivity: \( \eta = 1/\sigma \))

\[
\eta = \frac{1}{32 \pi \epsilon_0^2} \frac{e^2 m_e^{1/2}}{(k_B T)^{3/2}} \ln \Lambda
\]

\[
\Lambda = n_e \lambda_D^3 = n_e \times \left( \frac{\epsilon_0 k_B T_e}{n_e e^2} \right)^{3/2}
\]

only weak dependence of \( n_e \) in \( \ln \Lambda \)!

\( \rightarrow \mathbf{J} \) independent of number of charge carriers

From Helsinki University online lecture
Possible limitation: emittance degradation from non-uniform current

- Hotter plasma on-axis
- More current on-axis
- Stronger gradient on-axis (shorter focal length)
- Curved gradient $\Rightarrow$ emittance degradation

*Simulation by Bobrova*
On-axis current concentration reveals itself as donut mode when beam is over-focused.
On-axis current concentration reveals itself as donut mode when beam is over-focused.
Stability: 0.5% rms jitter in current at e-beam timing

At fixed timing,
2 Amps jitter @ 450 Amps
(0.5% rms)
for 500 consecutive shots
Active Plasma Lens:
• Strong gradients observed
• Ideal for compact & GeV applications
• Understand limits wakefields and non-uniform current
• More uniform for optimized timing, pressure, diameter, beam size
• Nov 2016: DESY collaboration at Mainz accelerator
End
70 Amps, uniform

48 Amps, actual

60 MeV

40 MeV
Effect of on-axis current (stronger near-axis B-field gradient)

\[ J(r) = J_0 \exp\left(-2r^2/w^2\right) \]

\[ \int_0^R 2\pi J(r) r dr = I_0 \Rightarrow J_0 = \frac{2I_0}{\pi w^2 \left[1 - \exp\left(-2R^2/w^2\right)\right]} \]

\[ 2\pi r B(r) = \mu_0 \int_0^r 2\pi J(r') r' dr' \Rightarrow B(r) = \frac{\mu_0 J_0 w^2}{4r} \left[1 - \exp\left(-2r^2/w^2\right)\right] \]